

THE MYSTERY OF THE WMAP COLD SPOT

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ABSTRACT

The first and third year data releases from the Wilkinson Microwave Anisotropy Probe (WMAP) provide evidence of an anomalous Cold Spot (CS) at galactic latitude $b = -57^\circ$ and longitude $l = 209^\circ$. We have examined the properties of the CS in some detail in order to assess its cosmological significance. We have performed a cluster analysis of the local extrema in the CMB signal to show that the CS is actually associated with a large group of extrema rather than just one. In the light of this we have re-examined the properties of the WMAP Internal Linear Combination (ILC) and co-added “cleaned” WCM maps, which have previously been used for the analysis of the properties of the signal in the vicinity of the CS. These two maps have remarkably similar properties on equal latitude rings for $|b| > 30^\circ$, as well as in the vicinity of the CS. We have also checked the idea that the CMB signal has a non-Gaussian tail, localized in the low multipole components of the signal by representing the CMB signal S outside the Galactic mask as a collection of signals for each equal latitude ring $s(b)$, $S = \bigcup s(b)$. For each ring we apply a linear filter with characteristic scale R , dividing the CMB signal in two parts: the filtered part, with characteristic scale above that of the filter R , and the difference between the initial and filtered signal. Using the filter scale as a variable, we can maximize the skewness and kurtosis of the smoothed signal and minimize these statistics for the difference between initial and filtered signal. We find that, unlike its Northern counterpart, the Southern Galactic hemisphere of the CMB map is characterized by significant departure from Gaussianity of which the CS is not the only manifestation: we have located a ring, on which there are “cold” as “hot” spots with almost the same properties as the CS. Exploiting the similarity of the WCM and the ILC maps, and using the latter as a guide map, we have discovered that the shape of the CS is formed primarily by the components of the CMB signal represented by multipoles between $10 \leq \ell \leq 20$, with a corresponding angular scale about $5 - 10^\circ$. This signal leads to modulation of the whole CMB sky, clearly seen at $|b| > 30^\circ$ in both the ILC and WCM maps, rather than a single localized feature. After subtraction of this modulation, the remaining part of the CMB signal appears to be consistent with statistical homogeneity and Gaussianity. We therefore infer that the mystery of the WMAP CS reflects directly the peculiarities of the low-multipole tail of the CMB signal, rather than a single local (isolated) defect or manifestation of a globally anisotropic cosmology.

1. INTRODUCTION

An extremely Cold Spot (CS), apparently inconsistent with the assumption of statistically homogeneous Gaussian fluctuations, was detected in a wavelet analysis (Vielva et al. 2004; Cruz et al. 2005; Cayon, Jin and Treaster 2005; Cruz et al. 2006) of the first-year data release from the Wilkinson Microwave Anisotropy Probe (WMAP). More recently, the existence of this spot has been confirmed by Cruz et al. (2007a) the WMAP third year data release (Hinshaw et al. 2007; Spergel et al. 2007). The WMAP CS is centered at the position $b = -57^\circ, l = 209^\circ$ in Galactic Coordinates and has a characteristic scale about 10° . As was pointed out by Cruz et al. (2006), the frequency dependence of the signal in the spot area is extremely flat. This fact has been used by the authors mentioned above to argue that the WMAP

CS belongs to the CMB signal, rather than any form of foreground emission. Cruz et al. (2006) pointed out that the reason the CS was not been detected in real space before the wavelet analysis was that it was hidden amongst structures at different scales.

As an apparent example of non-Gaussian behaviour in the WMAP CMB signal, the CS has attracted very serious attention from the theoretical point of view. Tomita (2005) suggested that the CS can be related to second-order gravitational effects. Inoue and Silk (2007) proposed a model involving local compensated voids. The origin of the CS in connection to the brightness and number of counts of the NVSS sources (smoothed on the scale of a few degrees) was recently discussed by Rudnick, Brown and Williams (2007). They have detected a 20-45% dip in the smoothed NVSS source counts which can be interpreted, they argue, as a manifestation of the integrated

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Sachs-Wolfe effect, seen for a single region of the CMB sky. Jaffe et al. (2005, 2006), Cayon et al. (2006), and McEwen et al. (2005, 2006) have investigated the Bianchi VII_h anisotropic cosmological model as a possible explanation of the CS and other features of the WMAP low multipoles. Recently, Cruz et al. (2007b) have pointed out that the CS could be produced by a cosmic texture, assuming that the CMB signal is a combination of the Gaussian and non-Gaussian parts. The present status of the problem of existence of the CS remains uncertain, despite the presence of a vast collection of theoretical suggestions. Nevertheless, if we believe that one particular part of the WMAP CMB signal contains non-Gaussian features, it would be necessary to seek corroborating evidence of non-Gaussianity elsewhere in order to understand their properties more fully. In this Paper we therefore present a detailed investigation of the properties of the CS, focusing attention on the following topics.

First, in Section 2, we show how the CS can be easily detected in the pixels domain not only in the derived CMB signal, but even in the WMAP maps for K-W bands before separation of the signal into CMB and foreground components.

Second, we will demonstrate that the CS belongs to a cluster of local minima, the spatial distribution of which is modulated by the large-angle modes of the CMB signal outside the Galactic plane. For that we use the Internal Linear Combination (ILC) III map and the co-added WCM map with $N_{side} = 512$ in the HEALPix format, converted to GLESP format (Doroshkevich et al. 2003), where each iso-latitude ring has the same number of pixels $N_\phi = 2048$ in azimuthal direction ϕ in polar coordinates. After that we perform a cluster analysis (Novikov and Jorgensen 1996) of the positive and negative peaks for selected rings in the area outside the Kp0 mask, mostly concentrating our attention on the ring crossing the CS at its extremum $b = -57^\circ$ and $-180^\circ \leq \phi \leq 180^\circ$. Taking into consideration the signal for each ring with the latitude b , we can investigate the morphology of the CMB signal at each latitude for the whole range of ϕ . This approach allows to connect the morphology of the CS to the signal outside the CS for the same latitude $b = -57^\circ$. We will show that the cluster contained the CS, is not a unique feature of the $b = -57^\circ$ iso-latitude ring. For example, close to the CS there are two significant clusters of maxima, but these peaks have lower amplitude than the CS.

Next, since the origin of large clusters of extrema is related to the angular modulation of the signal on large scales (see Novikov and Jorgensen 1996), we split the CMB signal into two parts. To do that we use the skewness and the kurtosis of the signal for selected rings, including the $b = -57^\circ$ ring. Then by using a simple linear smoothing filter with characteristic scale R we separate the signal into a smoothed component and to a difference between initial signal and the smoothed component. For the smoothed signal we define the skewness $S(R)$ and the kurtosis $K(R)$ as functions of R and find the scale of filtering which maximize both these characteristics $S(R_{opt}), K(R_{opt}) \rightarrow \max$. By using this scale R_{max} we separate the initial CMB signal in two parts, one of them (the smoothed one) contain a maximally non-Gaussian signal, and the another one (initial signal minus

the smoothed one) the maximally Gaussian signal. The non-Gaussian part is mainly formed by the signal localized at the range of multipoles $2 \leq \ell \leq 20$ and the other one belongs to the $\ell > 20$ multipoles. Our analysis clearly demonstrates that the pronounced non-Gaussianity of the CS reflects directly the existence of a large-scale angular modulation of the CMB signal with $10 \leq \ell \leq 20$.

Finally, using cluster analysis in combination with skewness and kurtosis statistics, we are able to detect a few additional cold and hot spots on the same $b = -57^\circ$ ring as the famous one. To show that the effect on clustering of the peak by low multipoles of the CMB is very common, we took into consideration the north Galactic hemisphere with deficit of the power and have found a few cold and hot spots. The idea of implementing of cluster analysis in combination with skewness and kurtosis statistics was stimulated by Vielva et al. (2004), Cruz et al. (2005, 2006, 2007a), Cayon, Jin and Treaster (2005), and especially Eriksen et al. (2004) and Larson and Wandelt (2005).

In our analysis we use both the WCM map and the ILC third year map, which are very similar outside the $b = \pm 25^\circ$ cut of the Galactic plane. For our analysis we use the high resolution ILC III map as a guide map to mark possible zones of the CMB sky in which the enhanced clustering of the peaks is expected to be considerable.

2. “NAIVE” DETECTION OF THE WMAP COLD SPOT

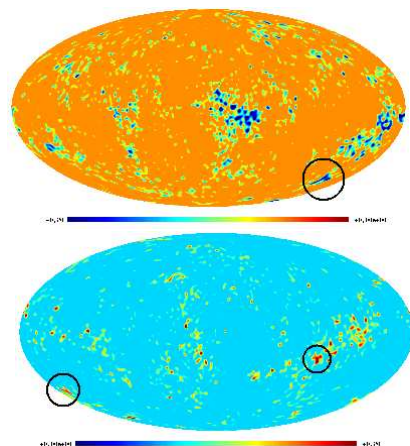


FIG. 1.— The map for negative (top) and positive thresholds $-0.2 \leq T \leq -0.05$ and $0.05 \leq T \leq 0.2$ of the ILC III map. For the top map the black circle marks the location of the CS in the Galactic coordinates. For the bottom map the black circles mark high amplitude positive peaks.

As was pointed out in the Introduction, historically the CS was detected in the WMAP data as one of the deepest minima of the CMB signal, using method based on wavelets. Our first aim is to show that this CS can actually be detected quite straightforwardly in the pixel domain using simple thresholding techniques. In Fig.1 we take two thresholds of the ILC III signal at the range of temperatures $-0.2 \leq \Delta T \leq -0.05$ and $0.05 \leq \Delta T \leq 0.11 \text{ mK}$ and map them with a color scale $-0.2, 0$ and $0, 0.2$, respectively.

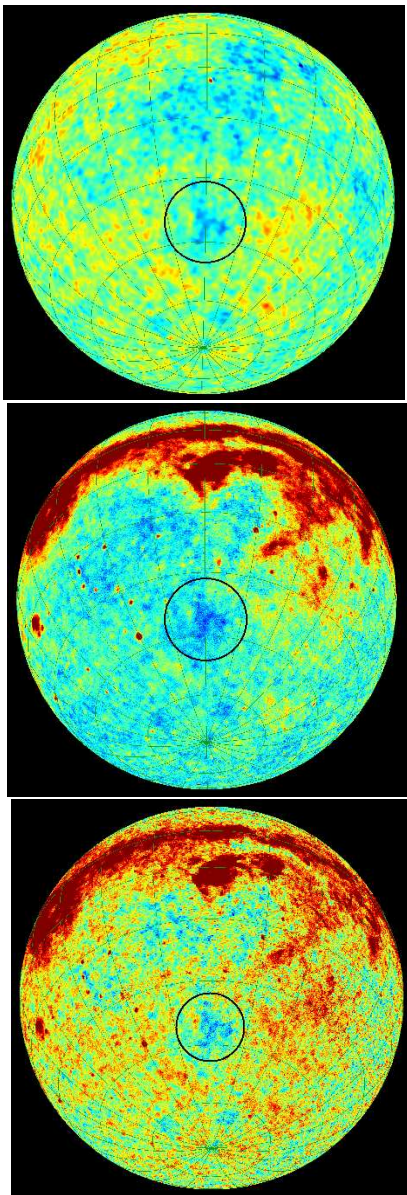


FIG. 2.— The SKY Viewer maps for the ILC III (top), the K and Ka bands of the WMAP (second and third from the top) map with cutoff of intensity down to the minimal values. For all the maps the zone of the CS has been placed at the center of the black circle.

From this figure one can immediately see the difference in statistical properties of the positive and negative thresholds of the map. First of all, it is important to note that a majority of the positive peaks are associated with the area around $|b| \leq 25 - 30^\circ$ (see the bottom plot), while the negative peaks are present well outside this region of the map. Secondly, the amplitude of the highest positive peaks is limited by $0.11mK$, while for the negative peaks it is about $-0.019mK$. For the CS the temperature of the negative peak is in agreement with the estimate of Cruz et al. (2007a). In Fig.2, by using the SKY Viewer, we map the ILC III K and Ka maps, in which the CS is located in the center of the area marked by the black circle (the blue cluster of peaks inside the black circle). From the K and Ka maps one can clearly see that the CS is clearly observed even without subtraction of the CMB signal. In agreement with Rudnick, Brown and Williams (2007) we

note that there is a cluster of negative peaks, rather than one single peak.

To show the local structure of the zone containing the CS, in Fig.3 we plot high resolution ($\ell_{max} = 100$) images of the inner structure of the CS zone, including the Haslam et al. (1982) map.

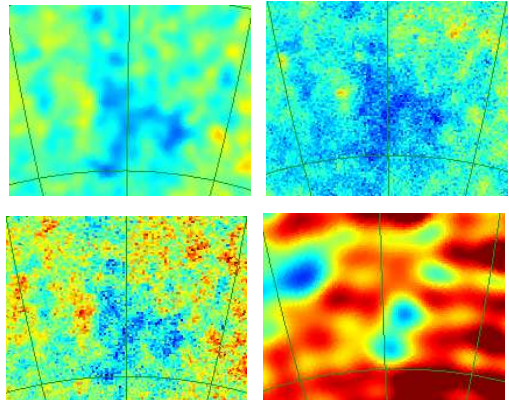


FIG. 3.— The CS zone image projection (in the pixel domain) of the ILC III, K, Ka and Haslam et al. (1982) maps. The size of all the maps is $25 \times 25^\circ$.

From Fig.2 and Fig.3 one can draw the important conclusion that the zone of the CS is surrounded by zones of hot spots, clearly seen in Fig.2 just on the right and left hand side of the CS.

If the origin of the CS is related to large-angular modulation of the CMB map and possible anisotropy of the power distribution across the sky, it would be naturally to expect that additionally to the WMAP CS detected by Vielva et al. (2004) and others, we could find other cold and hot spots similar or even equal morphological properties. To show that this is the case for WCM and the ILC III maps, in the next section we will use a cluster analysis of these maps in order to mark possible zones of peculiar distribution (clusters) in the signal.

3. CLUSTER ANALYSIS OF THE CMB MAPS IN THE VICINITY OF THE CS

The main idea behind our implementation of cluster analysis is to divide the CMB sky above and below the Kp0 mask into iso-latitude rings and then to analyze the properties of the signal for each ring separately. First, let us briefly describe the statistical properties of the signal $T(\theta = \theta_c, \varphi)$ for fixed latitude b . Let us take under consideration the distribution of peaks above and below some given threshold $\nu_t = \Delta T / \sigma_0$, where σ_0 is the square root of the variance σ_0^2 for each ring. For a one-dimensional cross-section of the CMB map we introduce a definition of a cluster of maxima considering, for example, two points, x_1 and x_2 ($x_1 < x_2$) from the ring. If for all the points in the interval $x_1 \leq x \leq x_2$ we have $\Delta T(x) > \nu_t \sigma_0$, we call these points x_1, x_2 connected to each other with respect to the threshold ν_t . A collection of maxima of ΔT located at the points $\{x_k\}, k = 1, 2, \dots, K$ we will call a cluster of length D if all the points $\{x_k\}$ are connected to each other when the threshold ν_t is applied⁶. The length of the cluster $D = |x_1 - x_2| / 2\pi$ is an analogue of the definition of the two-dimensional area (Cruz et al. 2007a), but in a one-dimensional case. For a random Gaussian field the

⁶ The same definition can be introduced for cluster of minima, when instead of maxima we will use the minima of T .

statistic of length of the clusters (one-dimensional) and the area (two-dimensional) are similarly sensitive to the spectral parameters of the random field, as described by Bardeen et al. (1986). However, the difference between cluster analysis and area statistics becomes obvious in application to non-Gaussian random fields. Let us assume for a moment that some zone of the signal is characterized by the area statistic, and that this has a very non-Gaussian value. To characterize the properties of the non-Gaussian field it seems to be very important to know, if this area is related to a uniform structure with one single peak, or formed from a cluster of peaks with the same area. This is why for our analysis of the WCM and the ILC III statistical properties we prefer to use cluster analysis (CA). In our analysis, in addition to the length of cluster D , we will use its size S which we define as the number of maxima (or minima) above (or below) the threshold ν_t within the interval length $D = |x_1 - x_2|/2\pi$.

3.1. Statistical properties of the signals for equal latitude ring

To describe the statistical properties of equal latitude rings we will use the approach proposed by Chiang and Naselsky (2007).

The standard treatment for a full-sky CMB signal $T(\theta, \varphi)$ is via spherical harmonic decomposition:

$$T(\theta, \varphi) = \sum_{\ell=0}^{\ell_{\max}} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_{\ell m}(\theta, \varphi), \quad (1)$$

where θ and φ are the polar and azimuthal angle, respectively, and $a_{\ell m}$ are the spherical harmonic coefficients. Here $Y_{\ell m}$ are the spherical harmonics, defined in terms of Legendre polynomials and plane waves:

$$Y_{\ell m}(\theta, \varphi) = N_{\ell m} P_{\ell}^m(\cos \theta) \exp(im\varphi), \quad (2)$$

where

$$N_{\ell m} = (-1)^m \sqrt{\frac{(2\ell+1)(\ell-m)!}{4\pi(\ell+m)!}}. \quad (3)$$

For a random Gaussian CMB sky the properties of the statistical ensemble of realizations are completely specify by the power spectrum

$$C(\ell) = \frac{1}{2\ell+1} \sum_{m=-\ell}^{\ell} \langle |a_{\ell m}|^2 \rangle, \quad (4)$$

while for each single realization we expect to find some deviation from $C(\ell)$ due to the ‘‘cosmic variance’’ effect. Below we will use a polar coordinate system in which the Galactic plane ($b=0$) is associated with $\theta = \pi/2$.

Let us analyse the signal $T(\theta_c, \varphi)$ from the equal-latitude ring at $\theta = \theta_c$, where $|\theta_c| \geq |\theta_{\text{mask}}|$, where $|\theta_{\text{mask}}|$ is the maximum latitude of any foreground masks. This ring $T(\theta_c, \varphi) \equiv T_c(\varphi)$ is a one-dimensional signal, for which we can construct a Fourier transform with coefficients g_m :

$$T_c(\varphi) = \sum_{m=-\ell_{\max}}^{\ell_{\max}} g_m \exp(im\varphi), \quad (5)$$

where

$$g_m = \int_0^{2\pi} d\varphi T_c(\varphi) \exp(-im\varphi). \quad (6)$$

We can then relate the ring to the full-sky signal via Eq.(2) and (5) and get

$$g_m = \sum_{\ell \geq |m|}^{\ell_{\max}} N_{\ell m} P_{\ell}^m(\cos \theta_c) a_{\ell m}. \quad (7)$$

That is to say that the Fourier coefficients g_m of the ring can be expressed as a combination of the full-sky $a_{\ell m}$. Defining the variance of the signal for equal latitude ring as

$$\text{Var}T = \frac{1}{2\pi} \int_0^{2\pi} d\varphi [T(\varphi) - \langle T \rangle]^2, \quad \langle T \rangle = \frac{1}{2\pi} \int_0^{2\pi} d\varphi T(\varphi) \quad (8)$$

and after substitution of Eq.(5) into Eq.(8) we have

$$\begin{aligned} \langle T \rangle &= \sum_{\ell} N_{\ell, m=0} P_{\ell}(\cos \theta) a_{\ell, m=0}, \\ \text{Var}T &= \sum_{\ell} \sum_{\ell'} \sum_{m \neq 0} N_{\ell, m} N_{\ell', m} P_{\ell, m}(\cos \theta) \times \\ &\quad \times P_{\ell', m}(\cos \theta) a_{\ell, m} a_{\ell', m}^*. \end{aligned} \quad (9)$$

For a random Gaussian field (GRF), after average over realizations, the combinations of the $a_{\ell m}$ coefficients in Eq.(9) satisfy the following conditions:

$$\langle a_{\ell, m} a_{\ell', m'}^* \rangle = C(\ell) \delta_{\ell, \ell'} \delta_{m, m'} \quad (10)$$

and then

$$\text{Var}T = \sum_{\ell} \sum_{m \neq 0} N_{\ell, m}^2 P_{\ell, m}^2(\cos \theta) C(\ell) = \sum_{\ell} W(\ell, \cos \theta) C(\ell) \quad (11)$$

where $W(\ell, \cos \theta) = \sum_{m \neq 0} N_{\ell, m}^2 P_{\ell, m}^2(\cos \theta)$ is the window function of the ring.

Our approach here is a special case for a well known theoretical prediction: any n dimensional cross sections of N dimensional Gaussian random signal produce a Gaussian signal as well. However, this general theory tells us about the GRF, represented as a sets of realizations. For one single realization of the CMB sky, as the WMAP signal does, the Eq.(11) is no longer available. More general, instead of Eq.(11) we have

$$a_{\ell, m} a_{\ell', m'}^* = C(\ell) G_{\ell, \ell'}^{m, m'}, \quad \langle G_{\ell, \ell'}^{m, m'} \rangle = \delta_{\ell, \ell'} \delta_{m, m'} \quad (12)$$

Thus,

$$\begin{aligned} \text{Var}T &= \sum_{\ell} \sum_{\ell'} \sum_{m, m' \neq 0} N_{\ell, m} N_{\ell', m'} P_{\ell, m}(\cos \theta) \times \\ &\quad P_{\ell', m'}(\cos \theta) C(\ell) G_{\ell, \ell'}^{m, m'}, \\ W(\ell, \cos \theta) &= \sum_{\ell'} \sum_{m \neq 0} N_{\ell, m} N_{\ell', m'} \times \\ &\quad P_{\ell, m}(\cos \theta) P_{\ell', m'}(\cos \theta) G_{\ell, \ell'}^{m, m'} \end{aligned} \quad (13)$$

The matrix $G_{\ell, \ell'}^{m, m'}$ describes the coupling between different modes ℓ, m and ℓ', m' for the random process $T(\varphi)$, which leads to variations of the variance $\text{Var}T(\theta)$ for different rings ($\theta = \text{const}$). One can see that integration of Eq.(13) over θ preserves spontaneous correlations ($G_{\ell, \ell'}^{m, m'} \neq \delta_{\ell, \ell'} \delta_{m, m'}$) even for the whole sky. However, for a particular ring we will have an additional modulation of these correlations depending on the θ through the window function $W(\ell, \cos \theta)$. In Fig.4 we show the dependence of

$VarT$ on the galactic latitude for the WCM and the ILC III maps.

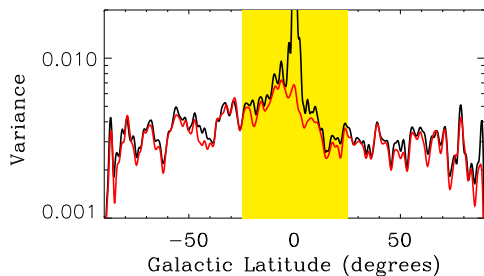


FIG. 4.— The variance for the WCM map (the black line) and the ILC III map (the red line) versus the galactic latitude.

First of all, we stress the remarkable similarity of these maps outside the region of the Kp0 mask. Moreover, the variance of the WCM map per each ring matches the variance of the ILC III almost exactly for all latitudes, except for the $|b| \leq 5^\circ$ zone. Secondly, note that the $b = -57^\circ$ ring lies near to the local maxima of the variance. The width of this zone is about $\theta \sim 10^\circ$. Thirdly, from Fig.4 one can clearly see the asymmetry of the variance for the rings located at the North and the South hemispheres. No special analysis is required to observe that the variance of the Southern hemisphere is higher than for the North. This result is in agreement with Eriksen et al. (2004) investigations of the asymmetry of the power per 10° patches of the sky, and Larson and Wandelt et al. (2005) analysis of peak statistics. Furthermore, looking at Fig.4 at $b = -25^\circ$ one can find the global maxima of the variance outside the Kp0 mask. This zone is clearly seen in Fig.1 just below the galactic plane as a large cluster of minima. Could the origin of this cluster be the same as for the cluster around the CS? To answer this question we need to look closely on the properties of the signal for the ring $b = -57^\circ$ in the azimuthal direction.

3.2. The clusters in the ring at $b = -57^\circ$

In this section we draw attention to the azimuthal distribution of the signal for the iso-latitude ring with $b = -57^\circ$, which contains the CS. In Fig.5 we plot $T(\theta_c, \phi)$ for WCM and the ILC III maps smoothed by the angle $\Theta_c = 1^\circ$. This figure clearly demonstrates that there are no significant differences of morphology of the CS in the ILC III and WCM maps. In the following analysis we change the reference system of coordinate from the Galactic one to one in which $\phi = 0$ is associated with the Galactic center and then all the values of ϕ are counted clockwise up to $\phi = 360^\circ$. The GLESP pixelization allow us to fix the same number of pixels for each iso-latitude ring ($N = 2048$) and consequently the location of each pixel $0 \leq k \leq N$ is related to the angle ϕ_k as $\phi_k = 2\pi k/N$.

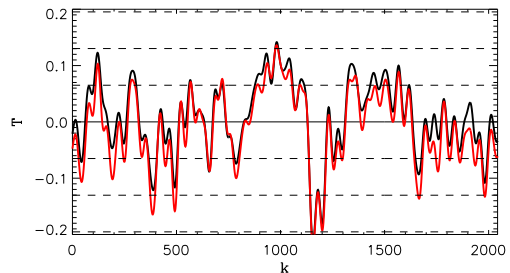


FIG. 5.— Plot of the one-dimensional ring through the center of the CS ($b = -57^\circ$) in azimuthal direction. The solid black line corresponds to the WCM signal and the red line is for the ILC III map. All the maps were smoothed by the angle $\Theta_c = 1^\circ$. For this ring $\sigma_0 = 0.0651 mK$. The dashed lines mark the thresholds $|\nu_t| = 1, 2, 3$.

In Fig.5 the CS is clearly seen at $k \simeq 1130 \div 1224$ as a cluster of minima (C10, $n=10$) of the signal in the WCM and ILC III maps. Moreover, there are no significant differences between these two signals either for the whole ring, or for the zone of the CS. Fig.6 shows the distribution of the length of the clusters versus the number of cluster. Clusters of maxima are shown in black, minima in red. As one can see from Fig.6 there are about 17 clusters of maxima and 17 clusters of minima. The mean length of the ILC and WCM clusters of maxima $\langle D_{ilc}^+(n) \rangle \simeq 0.0337$, while for the clusters of minima we get $\langle D_{ilc}^-(n) \rangle \simeq 0.0256$. For comparison, after taking 1000 realizations for the GRF with the WMAP best fit Λ CDM power spectrum we have found $\langle D_{grf}^+(n) \rangle \simeq 0.0303$ and $\langle D_{grf}^-(n) \rangle \simeq 0.0315$.

The cluster C10 containing the CS has length $D_{CS} = 0.0459 \simeq 1.5 \langle D_{grf}^-(n) \rangle$ and its dimension is $D^- = 2$ at $\nu_t = 0$. At the same time $D_{CS} / \langle D_{ilc}^-(n) \rangle \simeq 1.8$ at $\nu_t = 0$. These ratios tell us that this cluster looks like an “ordinary” cluster defined by the threshold $\nu_t = 0$ without any significant deviation from the statistical properties of a random Gaussian Field. However, the extraordinary properties of this cluster become obvious when we take into consideration other thresholds, $-3 \leq \nu_t \leq -2$. From Fig. 6 we see that even for this range of ν_t the cluster still appears as a cluster of minima at the level $\nu = -3$, and local maxima at $\nu_t = -2$. Novikov and Jorgensen (1996) have pointed out that for the cluster of minima with $d^- = 2$ the conditional probability $P(k_1, k_2)$ to find two negative peaks located at the points k_1 and k_2 in this cluster is less than the conditional probability to find one maxima in between these points with height above ν_t . In our case this threshold is $\nu_t \simeq -2$ and $P(k_1, k_2) \leq 0.02$.

However, this cluster is not a unique feature of the $b = -57^\circ$ ring. There are two clusters of maxima (see Fig.5), namely the cluster C9 with $k_{min} = 825$ and $k_{max} = 1129$ with $D_{C9} = 0.15 = 4.40 \langle D_{ilc}^+(n) \rangle$ and the cluster C11 with $k_{min} = 1310, k_{max} = 1634$, and $D_{C11} = 0.15 = 4.69 \langle D_{ilc}^+(n) \rangle$. Here k_{min}, k_{max} mark the coordinates of the cluster at $\nu_t = 0$ in the pixel domain. These clusters seen in Fig.6 are near to the cluster of minima C10 within which the CS lies.

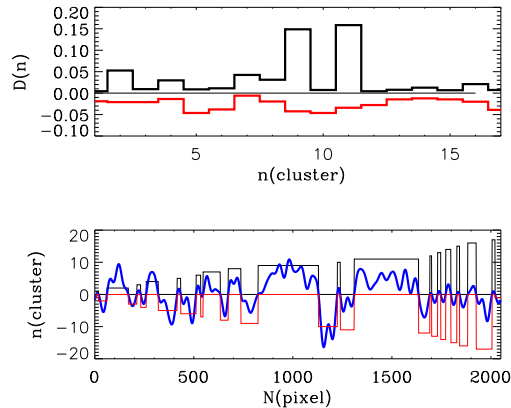


FIG. 6.— Top. The length $D(n)$ of the clusters $\nu_t = 0$ versus the number of cluster. The black line is for clusters of maxima, the red line represents the minima. Bottom plot. Number of cluster versus its position in the pixel domain (2048 pixels correspond to the 360° in azimuthal direction). The blue line corresponds to a one-dimensional ring of the ILC III map trough the CS in units of $5 * T_k / \sigma_0$. The negative sign for $D^-(n)$ marks the length of the clusters of minima.

The existence of such clusters with lengths above $4\langle D_{\text{ilc}}^+(n) \rangle$ and the cluster of minima C10 with such high amplitude (negative) peaks seems to be a quite peculiar feature of the $b = -57^\circ$ ring. However, these peculiarities are not so specific. To show that “over-clustering” of the CMB sky is a typical feature of the signal let us examine, for example, the ring with $b = +57^\circ$, located symmetrically to the $b = -57^\circ$ ring in respect to the Galactic plane. In Fig.7 we show $T(k)$, and the distribution of clusters versus their length for the ring with $b = 57^\circ$. This signal reveals a remarkable similarity of the morphology to the ring $b = -57^\circ$. As for the ring with CS, the ring $b = 57^\circ$ is characterized by very high level of clusterisation, the existence of the 3σ minima, as a member of the cluster with $D^- = 4$, and the existence of the positive cluster with $D^+ = 6$. Moreover, one can see that the cluster of maxima C7 at $k_{\min} = 547, k_{\max} = 633$ has a structure similar to the structure of the signal in the CS, but now for the maxima.

At the end of this section we would like to point out that the existence of clusters with the length $D > 3 \div 4\langle D \rangle$ is a quite rare event for the GRF. The presence of three clusters with $D \sim 3\langle D \rangle$ for single iso-latitude ring at $b = 57^\circ$, and two clusters with $D > 4\langle D \rangle$ for $b = -57^\circ$, as the CS as well, allow us to conclude that the ILC III and WCM maps are generally “over-clustered”. Attention on the CS was focused mainly because of the *amplitude* of the signal in that position. A more specific feature of the CS is not this, but that it is really a cluster of peaks with nearly the same amplitude, and there is a very large cluster near to it.

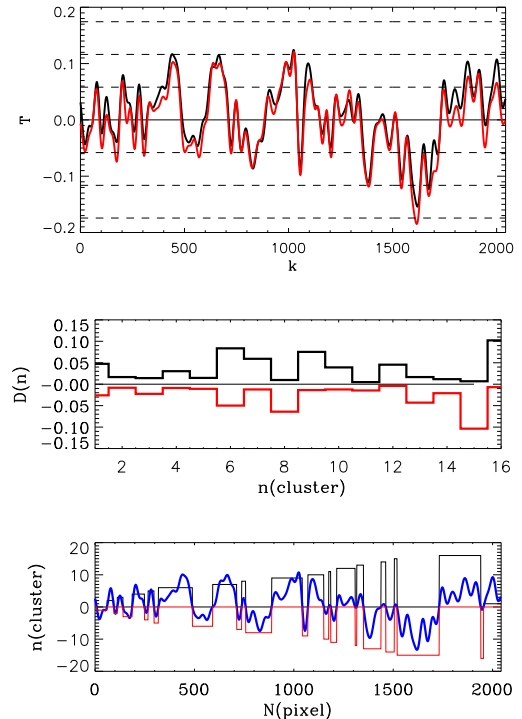


FIG. 7.— The same as for Fig.5 and Fig.6, but for the ring with $b = 57^\circ$.

4. “DE-CLUSTERING” OF THE ILC III AND WCM BY LINEAR FILTRATION

In this Section we re-examine an idea of Cruz et al (2007a), that the origin of the CS can be explained if the WMAP CMB signal contains non-Gaussian components in combination with the Gaussian ones. This idea seems to be quite natural since the low multipole part of the WMAP ILC III map reveals significant peculiarities of the signal: alignment between quadrupole and octupole, coupling with the Galactic foregrounds, low power of the quadrupole, and so on. Note that the WMAP team performed the analysis of Gaussianity of the CMB signal by subtracting the ten multipole components of lowest order from the map and claimed that the rest of the signal is in agreement with Gaussian statistics. However, asymmetry of the power of the CMB, discovered by Eriksen et al. (2004), the existence of the CS and another peculiarities, mentioned in Lasenby et al. (2007) and McEwen et al. (2007), raise the question of whether there is some mark imprint non-Gaussian features of the CMB, mainly localised in the low multipole range of the power spectrum.

If the WMAP CMB outside the Galactic mask $|b| = 25^\circ$ is the sum of Gaussian and non-Gaussian components, it seems natural to guess that these two signals would have different characteristic scales. The idea, which we will develop below, is to use a linear filter of the CMB signal for each ring of the map with variable scale of filtration R , which can divide the CMB signal in two parts: $S(k) = \overline{S}(k, R) + s(k)$, where $s(k)$ corresponds to Gaussian component and $G(k) = \overline{S}(k, R)$ corresponds to non-Gaussian one. Note that Cruz et al. (2007) use the wavelet approach to perform this analysis. We will instead use the

running window filter defined in the pixels domain as

$$\begin{aligned}\bar{S}(k, R) &= \langle T_k \rangle = \frac{1}{R} \sum_{j=0}^{R-1} T_{k+j-\frac{R}{2}}, \\ j &= \left[\frac{R-1}{2}, \dots, N_{pix} - \frac{R+1}{2} \right] \\ s_k &= T_k - \langle T_k(R) \rangle.\end{aligned}\quad (14)$$

Note that the choice of the linear filter is not so important for the criteria of separation of the Gaussian and non-Gaussian tails of the signal. One can for example use the Gaussian filter $\bar{S}(k) \propto \sum_j T_j \exp(-(k-j)^2/R^2)$, or any another reasonable filter. What is important is the chose of the criteria of the scale R . Following Cruz et al. (2006), we will use the skewness and kurtosis of the signals s_k as a functions of R trying to minimize the difference between their actual values and most probable values for the GRF.

In Fig.8 we show the skewness and kurtosis for the ILC III ring $b = -57^\circ$ for different scales of filtering R .

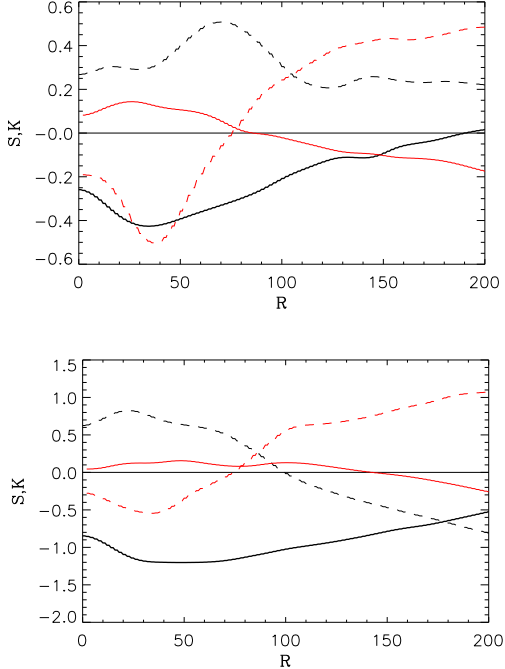


FIG. 8.— Top. The skewness and kurtosis of the ILC III $b = -57^\circ$ ring. The solid black and the dash black lines corresponds to the skewness and kurtosis of the smoothed part of the ILC III signal $\bar{S}(k, R)$. The red lines are for the signal s_k . Bottom. The same as top, but for the zone of the ring $k_{min} = 825, k_{max} = 1634$.

The skewness and kurtosis are very sensitive to the choice of the zone of the ring. For example, if we were to look at the zone $k_{min} = 825, k_{max} = 1634$ occupied by the clusters C^{+9}, C^{-10}, C^{+11} we get the skewness and kurtosis shown in Fig.8 (bottom plot). From these two plots one can see that the value of the parameter $R \simeq 70$ is preferred for this analysis. For this scale of filtering the kurtosis of the smoothed signal reaches a maximum, while for the Gaussian tail of the total signal both the characteristics are close to zero both for the whole ring and for the particular zone around the CS. In Fig.9 is shown the ILC III signal before (the red dotted line) and after subtraction of the smoothed over $R \simeq 70$ pixels (the black line). One

sees that the CS is eliminated.

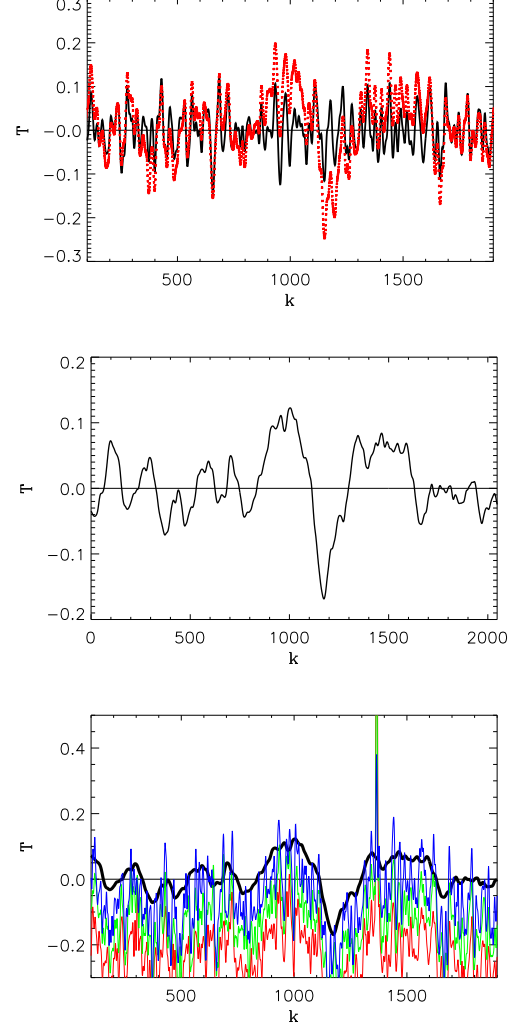


FIG. 9.— Top. The ILC III ring $b = -57^\circ$ before demodulation (red dots) and after (black line). Middle. The ILC III signal, smoothed over $R = 70$ pixels (the corresponding angular scale $\Delta\phi = 12.3^\circ$). Bottom. The smoothed ILC signal (thick black line) in comparison with signals from Ka (red line), Q (green line) and V band (blue line).

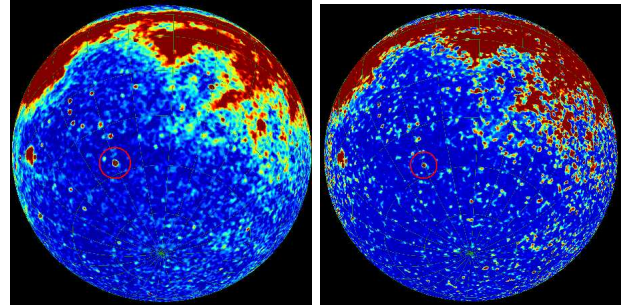


FIG. 10.— The Ka (left) and W band signal from the South hemisphere. The red circle marks the location of the point source at the edge of the CS.

However, from the middle plot of Fig.9 we can see that the CS, as well as all the large clusters come from the smoothed signal, which preserves all the non-Gaussian features mentioned in the previous section including the CS. The bottom plot of Fig.9 show one more significant peculiarity of the CS zone. Just at the edge of the CS one can

see a bright point source. The amplitude of this source is about $50\sigma_{cmb}$ for the K band and its drops down to $6\sigma_{cmb}$ for V and W bands. In Fig.10 we show this point source for the Ka and W bands. For the ILC III and WCM maps the residuals from this point source are associated with local maxima at the level of $1\sigma_{cmb}$.

5. ARE THERE OTHER “COLD” OR “HOT” SPOTS ?

This question seems to be one of the most important question of the entire analysis of the origin of the CS. Do we have the only one single peculiar zone in the CMB map, or are there more zones with non-Gaussian properties of the signal ? To answer this question, we suggest using the distribution of the variance of the CMB signal versus the latitude, shown in Fig.5. Let us focus attention on the Southern hemisphere and in particular on the point of maxima of the variance. We can see the following coordinates of the points of maxima: $\theta = -80^\circ, -70^\circ, -57^\circ, -30^\circ$. For these rings we performed the same analysis as for the ring at $b = -57^\circ$. All of them show the existence of non-Gaussian features, which can be identified by the same method as we have used for the $b = -57^\circ$ ring. In Fig.11 we show the signal for the ring -70° . A non-Gaussian hot spot clearly seen at $k \sim 850$ (the corresponding Galactic coordinates are $b = -70^\circ, \phi = 149^\circ$). The amplitude of the peak is about $4.8\sigma_\phi$, and is the member of the cluster with $d^+ = 6$ for $\nu_t = 0$. From Fig.11 one can obtain the optimal size of the filter scale $R \simeq 50$.

Fig.12 shows the same characteristics of the signal as the previous one, but for the $b = -30^\circ$ ring. Once again, the characteristic scale of filter is about $R = 70$ pixels. One may continue the search for other rings belonging to the Southern hemisphere, just by following the distribution of maxima of the variance from Fig.5. However, the question is could we find the same peculiarities of the signal for the Northern hemisphere, where a deficit of the variance occurs? In Section 3 we already mentioned that the ring $b = 57^\circ$ does indeed reveal over-clustering. Let us look closely at the skewness and kurtosis for that ring and find out the characteristic scale of non-Gaussianity. What is interesting is that for the $b = 57^\circ$ ring the skewness and kurtosis shown in Fig.13 are very close to the Gaussian characteristics, while for the zone $k_{\min} = 1520, k_{\max} = 1940$ with clusters $C^-15, k_{\min} = 1520, k_{\max} = 1731$ and $C^+16, k_{\min} = 1732, k_{\max} = 1940$ we can clearly see two minima for the demodulated signal, one at $R = 20$ and another at $R = 70$.

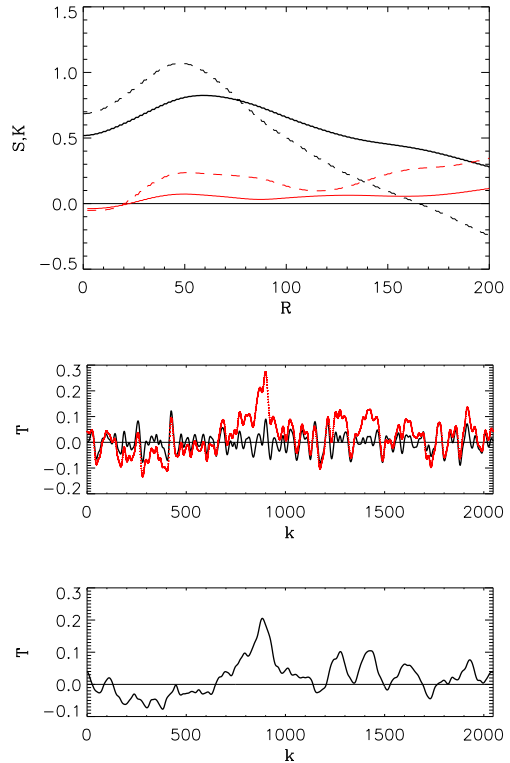


FIG. 11.— Top: The skewness and kurtosis for the whole $b = -70^\circ$ ring. The definition of the lines is the same as in Fig.9(top). Middle: The ILC III ring $b = -70^\circ$ before demodulation (the red dots) and after (the black line). Bottom: The ILC III signal, smoothed over $R = 50$ pixels (the corresponding angular scale $\Delta\phi = 8.8^\circ$.)

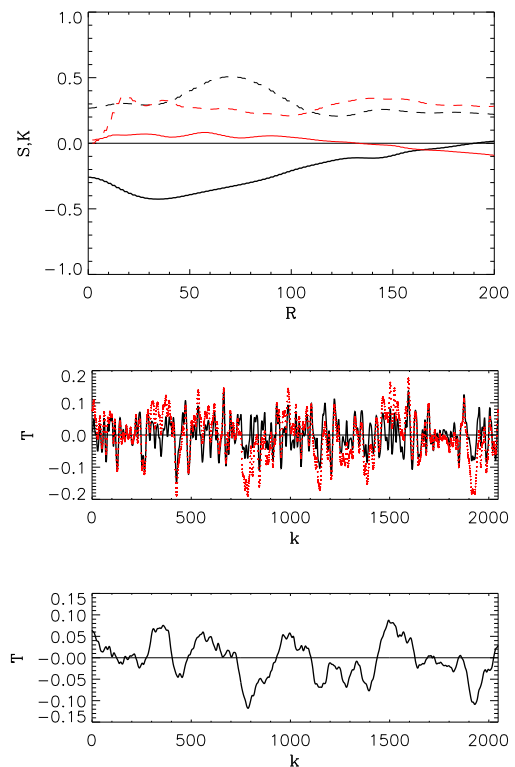


FIG. 12.— The same as Fig.9, but for $b = -30^\circ$ ring. The scale of filtration is $R = 70$ pixels.

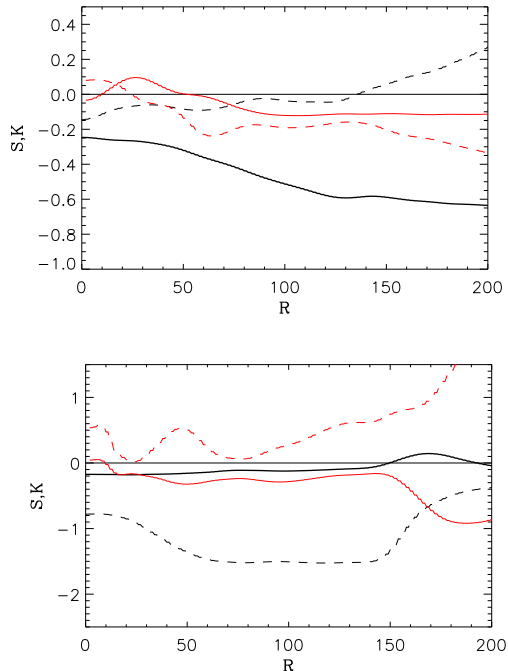


FIG. 13.— Skewness and kurtosis for the $b = 57^\circ$ ring. Top plot show these characteristics for the whole ring, bottom plot is for the combination of the clusters $C^-15 + C^+16$. The color of the lines is the same as for Fig. 9.

However, we stress that for the whole ring at $b = 57^\circ$ even without filtering, the skewness and kurtosis are close to the GRF, unlike, for example $b = -57^\circ$ ring. Moreover, increasing the scale R results in greater departures from Gaussianity, as it seen in Fig. 13. This tendency is common for $b = 72^\circ$ ring (the point of local minima of the variance), as for the $b = 78^\circ$ (the point of local maxima of the variance from Fig. 5). Thus, significant non-Gaussianity of the ILC III and WCM maps mainly corresponds to the South hemisphere, and is associated with large angular scales, around $9 - 12^\circ$.

To show that clustering of the extrema of the ILC III and WCM signals is a typical feature of the morphology, we show in Fig. 14 the the ILC III map seen from the North and the South Galactic poles. From the ring with the latitude $b \simeq 80^\circ$ one can easily find a big cluster of maxima and one big cluster nearly at the same latitude. For the South pole the zone of the CS connect with three big clusters of maxima.

6. COMPARISON WITH PECULIAR ZONES DETECTED BY WAVELETS

Detection of peculiarities in the CMB sky is obviously one of the major steps in the investigation of the departure of the signal from statistical isotropy and homogeneity. As was mentioned in the introduction, this problem was discussed by McEwen et al. (2006b), where Spherical Mexican Hat Wavelet (SMWH), Elliptical SMHW, and Spherical Butterfly Wavelet (SBW) approaches were applied to the detection and location of the position of the non-Gaussian spots in the CMB sky. Below we refer to these methods collectively as wavelet methods. McEwen et al. (2006b) summarize the detection of peculiar zones with approximate (estimated) coordinates of these zones.

We present these data in Table 1 in order to compare these results with our results of detection of non-Gaussian zones by implementation of cluster analysis.

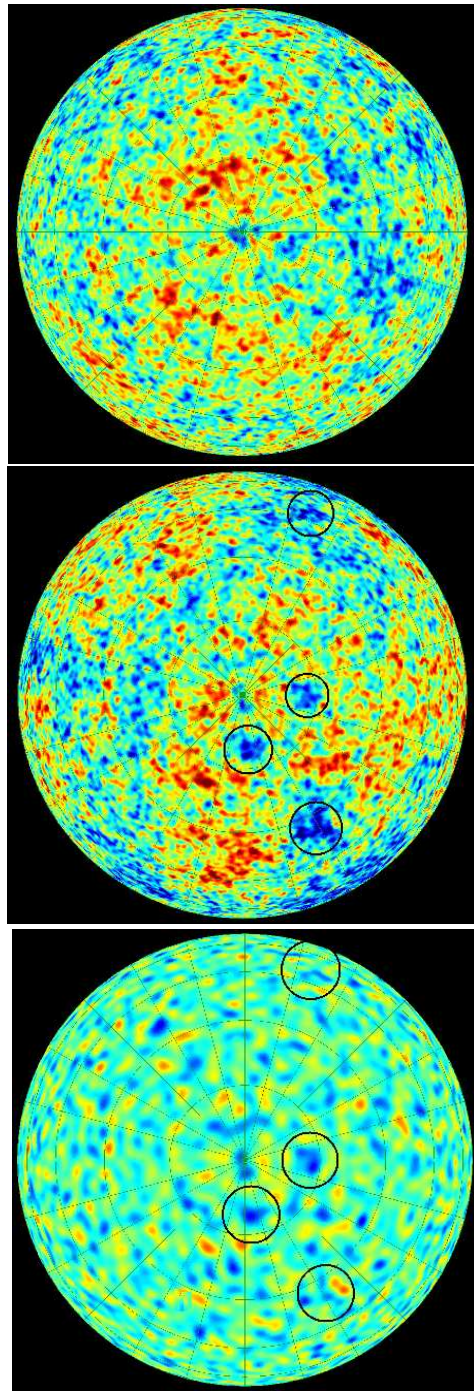


FIG. 14.— The image of the sky seen from the North and the South galactic poles (top and middle panels). The black circle marks the position of the CS. Bottom panel show the South hemisphere of the CMB sky after subtraction of the first 20 multipoles of the ILC III signal.

TABLE 1

THE PECULIAR ZONES OF THE CMB SKY DETECTED BY WAVELETS (FIRST AND SECOND COLUMNS) FROM MCEWEN ET AL. (2006B). THE SECOND COLUMN SHOWS THE LONGITUDE ϕ AND THE LATITUDE θ OF EACH ZONE. THE THIRD COLUMN SHOWS THE LONGITUDES OF THE CLUSTERS WHERE THE WAVELET PECULIAR POINT IS THE MEMBER. THE FORTH COLUMN SHOWS THE NUMBER OF EXTREMA IN THE CLUSTER (S) AND THE LENGTH OF THE CLUSTER (D). THE SIGN “—” MEANS THAT THE WAVELET PECULIAR ZONE IS NOT DETECTED BY CA WITH $\nu_t = 0$. STARS REDIRECT THE READER TO FIG. 15.

Zone	Location, W ϕ, θ (deg)	Location, CA ϕ_{min}, ϕ_{max}	S, D
1	75, 57	56, 86	3, 0.083
		157, 183	3, 0.072
		267, 304	4, 0.102
		304, 341	4, 0.102
2	75, 53	69, 93	2, 0.067
		40, 66	4, 0.072
		160, 183	3, 0.064
		273, 310	4, 0.103
3	323, 56	304, 339	4, 0.098
4	321, 62	278, 304	3, 0.072
		306, 333	2, 0.075
		333, 357	3, 0.067
5	267, 50	272, 307	4, 0.097
		251, 274	3, 0.064
		58, 82	3, 0.067
6	268, 45	313, 344	4, 0.086
		—	—
		129, 156	3, 0.075
7	213, 40	—	—
		30, 58	4, 0.078
		233, 258	4, 0.069
8	223, 30	—	—
		9, 37	4, 0.078
		136, 157	4, 0.056
9	160, 26	—	—
		174, 213	6, 0.108
		4, 32	4, 0.078
10	94, -28	—	—
		35, 62	3, 0.075
		142, 235	15, 0.258
11	81, -34	333, 352	4, 0.056
		—	—
		141, 181	7, 0.11
12	118, -42	187, 230	6, 0.12
		261, 294	7, 0.092
		—	—
13	20, -48	8, 30	4, 0.061
		333, 360+8	5, 0.097
		—	—
14	34, -31	7, 30	4, 0.058
		70, 105	4, 0.097
		243, 278	4, 0.097
15	230, -68	32, 55	5, 0.064
		288, 330	6, 0.117
		—	—
16	204, -56	220, 253	2, 0.092
		19, 52	2, 0.092
		63, 117	5, 0.15
17	186, -54	198, 216	2, 0.05*
		148, 188	4, 0.11
		223, 270	5, 0.13
18	218, -33	193, 216	2, 0.063*
		150, 193	4, 0.12
		224, 294	6, 0.194
		187, 230	3, 0.12
		258, 294	6, 0.10*
		—	—

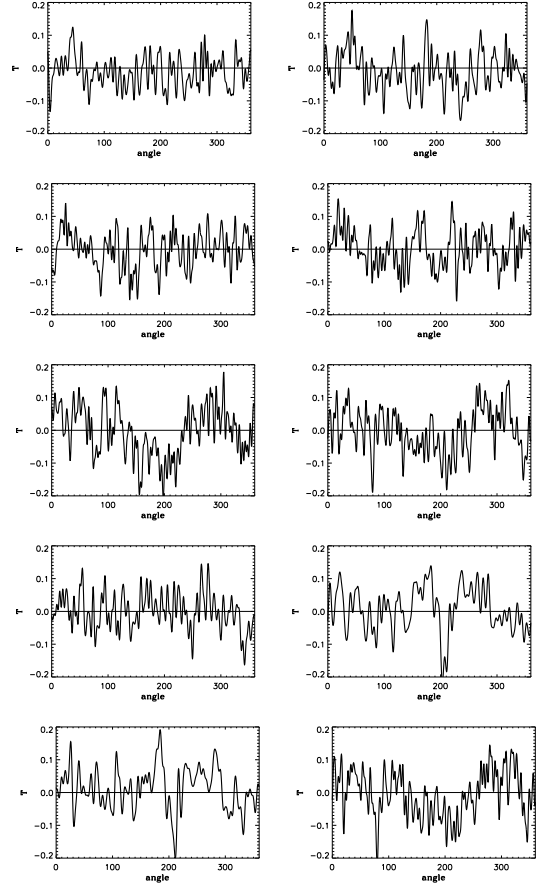


FIG. 15.— The CMB signal for equal latitude rings, marked in Table 1 by sign “—”. From the top to the bottom and from the left to the right are the zones 6-12, 16-18.

Let us discuss the properties of the signal for Zone 6 of Table 1. The wavelet approach gives us the azimuthal coordinate of the non-Gaussian spot $\phi_6 = 268^\circ$. From the top left plot, we can see that this point corresponds to a relatively small negative peak. The same morphology of the peaks is clearly observed at $\phi \sim 210^\circ$, and $\phi \sim 330^\circ$. Both peaks are very similar to the CS zone, but with smaller amplitudes. These two peaks probably contribute to the overall non-Gaussianity of the signal more significantly than the $\phi_6 = 268^\circ$ peak (see Table 1).

The next zone, which was not detected by cluster analysis at $\nu_t = 0$, is Zone 7 shown on the top right plot. According to the wavelet analysis, the azimuthal coordinate for this zone is $\phi_7 = 213^\circ$. Once again, here we have a negative peak, but the peak at $\phi_7 \simeq 240^\circ$ reveals a more significant departure from Gaussianity, it being a member of the cluster with number of minima $N = 4$.

For Zone 8, the wavelets analysis gives us $\phi_8 = 223^\circ$. We show the corresponding ring in Fig. 15 (second from the top left plot). The detected zones manifest themselves as points of maxima, while the negative peaks detected by cluster analysis, listed in Table 1 reveal significant departure from Gaussianity. For example, the cluster of minima at $\phi \simeq 140 - 160^\circ$ can be detected not only for $\nu_t = 0$ threshold, but even for $\nu_t = -2$, as a cluster of maxima and minima. Roughly speaking, all the pictures, shown in Fig. 15 clearly demonstrate that implementation of cluster analysis with different thresholds ν_t allow us to detect not

only single non-Gaussian zones, but also clusters of these zones which have non-local structure. To illustrate this tendency, we would like to draw attention to the bottom right picture belonging to the Zone 18. According to Table 1, the non-Gaussian zone is located at $\phi_{18} = 218^\circ$ being the member of the cluster and detected both by wavelet and by cluster analysis. However, there is another zone at $\phi \simeq 80^\circ$ with a morphology similar to the morphology of the CS. Moreover, looking at the shape of the signal at $\phi > 130$ one can see the modulation of the signal by low frequency harmonics. This type of non-Gaussianity is an argument in favor of hypothesis that low multipoles of the CMB signal are highly non-Gaussian.

7. CONCLUSION

We have re-examined the properties of the Internal Linear combination WMAP CMB map and the co-added WCM map by an analysis of the properties of the signal in the vicinity of the CS. These two maps of the CMB signal display remarkably similar structures on equal latitude rings at $|b| > 30^\circ$. We have re-examined the properties of the CS at the galactic latitude $b = -57^\circ$ and longitude $l = 209^\circ$ and shown that it is associated with the cluster with length $D \sim 3\langle D(n) \rangle$. In addition to the CS, we have also found a few more zones of the CMB signal with almost the same morphology, at $b = 57^\circ$, $b = -80^\circ$, $b = -30^\circ$.

From an analysis of the ILC III map we have shown that the shape of the CS is formed primarily by the CMB signal localized in multipoles between $10 \leq \ell \leq 20$ (corresponding to angular scales about $5 - 10^\circ$), in agreement with Cruz et al. (2005, 2007) results. Taking into account that the same modes lead to a modulation of the whole CMB sky, we subtracted these modes from the CMB signal. The demodulated CMB signal looks like a random one without

significant over-clustering.

We have investigated the asymmetry of the variance for iso-latitude rings in respect to the Galactic plane. The South hemisphere has excess variance in comparison to the North hemisphere. This is why local defects and large clusters, including the CS and its associated cluster, are mainly concentrated in the Southern hemisphere.

Taking all these investigations together, we believe that the mystery of the WMAP CS directly reflects directly the peculiarities of the low-multipole tail of the CMB signal, rather than a single local (isolated) defect or a manifestation of a globally anisotropic model. This interpretation does not preclude the possibility of an exotic origin of the CS and related phenomena, but it does specify more precisely what properties such explanations must generate. A satisfactory model of the CS must explain the entire range of its behaviour rather than only one aspect.

Our final remark is related to the definition of significance of the CS detection by different methods, based on the assumption that Gaussian statistics apply to the observed CMB sky. Ever since Eriksen et al. (2004), showed that the distribution of the power of the CMB across the sky is very anisotropic at the scales about 10° , it has been clear that Gaussian statistics are no longer a valid reference for determining the significance of this feature. Our approach to the large-scale angular modulation of the CMB is a possible alternative approach to this issue.

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⁷ <http://lambda.gsfc.nasa.gov>